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TO A PRESELECTED LANDING SITE

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SUMMARY

A two impulse maneuver allows great flexibility in the choice of a landing site. This paper assumes that a nominal two impulse abort trajectory has been selected before flight but that the velocity required to attain that abort trajectory from off nominal conditions will be computed manually on board the spacecraft. The method presented here accomplishes the entire plane change with the first impulse and restricts the application of this impulse to certain preselected ranges. The basic theory is valid for any transfer, in either planocentric or heliocentric space, between two positions whose inertial coordinates are known at prescribed times. Consequently, the theory would allow the method to be used for computing aborts from inbound trajectories, but reasonable fuel limitations restrict its primary applicability to outbound trajectories.

The computational procedure developed obtains three orthogonal components, in inertial axes, of the velocity to be gained for the first impulsive rocket firing and obtains the magnitude of second impulse. It is shown that the method is feasible for manual calculations, using four figure accuracy, with a family of graphs to match the time constraint.

This computational procedure can serve as a backup scheme for navigation in interplanetary space. Midcourse corrections can be calculated by this method.

INTRODUCTION

It is necessary that the crew of a spacecraft have a completely self-contained capability to accomplish an abort. There are at least two reasons for this requirement. The first is that communication difficulties of several hours duration may occur, consequently, the astronauts cannot rely on ground facilities to quickly process the current data and make a timely decision for them. The second reason is the possibility of error in either the ground computation or in the communication of the data to the spacecraft. The astronauts must be able to independently process the current data on board accurately enough to detect gross errors in the instructions received from the ground.

Most studies of the guidance problem of midcourse aborts have considered only one rocket firing and have been restricted to return trajectories which nominally lie within the plane of the initial trajectory. These two restrictions were adopted because they are approximately correct for the time critical abort, that is, a minimum time return without concern for a landing site, and because these restrictions simplify the analysis. In some abort situations, the selection of a landing site may be more important than the return flight time. Consequently, two-impulse returns and out-of-plane returns should be considered (ref. 1).

If a one-impulse abort maneuver is planned, there is a high probability that a substantial (greater than 10 m/sec) second velocity increment will be required to correct the aiming errors of the first velocity increment (ref. 2). Necessarily then the two-impulse capability will be present in the actual hardware. Since a second maneuver will generally be required, it is of interest to consider how a more complete utilization of this maneuver can increase landing site availability.

The relative flexibility of single-impulse and two-impulse aborts may be compared by examining a typical example. Suppose a vehicle whose entry range (from an altitude of 400,000 ft to touchdown) is 2000 nautical miles (3704 km) must abort at a distance of 150,000 km from the earth. Suppose also, that heating considerations limit the reentry speed such that the eccentricity of the entry trajectory must be less than unity. In addition the vehicle must meet entry corridor constraints (vacuum perigee = 6430 km). Under these conditions the single-impulse maneuver has a range of achievable incremental true anomaly between the abort point and the landing site of 175° to 225°. Under these same conditions, with two impulses, there are no theoretical bounds. Reasonable fuel limitations, however, would constrain the upper limit to about 275° and allow some improvement in the lower limit. Even with these restrictions the two-impulse maneuver has at least a 100 percent improvement in earth surface availability.

The substantial additional flexibility in landing site selection obtained by planning on appreciable fuel for the second rocket firing could lead to a major reduction in ground standby forces in the next decade. As space travel becomes more reliable, it is probable that ground standby forces will not be used at all and flexibility in abort landing site selection will be increasingly desirable.

The objective of this study then is to develop a method whereby astronauts can have an increased on-board flexibility in returning to a favorable landing site during an abort maneuver. The method must be sufficiently straightforward that the on-board computation can be accomplished in reasonable time without using a complex computer. This specification requires that every effort be made to simplify the description of the mechanics involved.

The extra computational complications of the two-impulse maneuver should be handled without burdening the on-board computer. Consequently, a procedure has been developed which primarily uses manual computations and a graph of certain elliptical orbit characteristics to determine three orthogonal components of the velocity increment for the first firing and the magnitude of the second velocity increment.

Because of the propagation of the errors of the first firing, the components of the velocity to be gained at the second firing must be determined near the time of the second firing. Since the method presented in this report makes the plane change at the first firing, this determination can be done by any simple system (such as those described in refs. 3 and 4) whose only object is to insure a safe return. Such a simple system will permit safe entry with only minor maneuvering required to attain the landing site safely. If available, the normal on-board navigation system or ground tracking could also be used to determine the second maneuver.

NOTATION

a	semimajor axis
ANG	angle from the abort position vector downrange to the landing position vector, deg
AOP	angle between initial and final orbital plane, deg
e	eccentricity of conic trajectory
h	angular momentum, km^2/sec
E	eccentric anomaly, rad
R	magnitude of the position vector from the center of the earth to the spacecraft, km
$\overline{R}$	position vector, km
T	time, sec
V	velocity magnitude, km/sec
$\overline{V}$	velocity vector, km/sec
$\overline{U}$	unit vector
θ	true anomaly, rad
μ	gravitational parameter of the earth (3.98603×10^5 km^3/sec^2)
ΔV	incremental velocity magnitude, km/sec

Subscripts

A	incremental change of a variable on the transfer trajectory
B	incremental change of a variable on the approach trajectory

C	incremental change of a variable between perigee and landing
D	downrange component, perpendicular to the radial component
L	landing site
P	perigee
R	radial component
T	total
0	conditions just before the first velocity impulse
req	conditions just after the first velocity impulse
β	conditions just before the second velocity impulse
F	conditions just after the second velocity impulse
1	first velocity impulse
2	second velocity impulse

GENERAL CONSIDERATIONS

After a decision to abort has been made, the astronauts must select a landing site and a satisfactory abort trajectory which reaches the site at the desired time. However, the selection of an abort trajectory is affected by many factors, including the following:

1. The present range from earth.

2. The time of landing, which must be known in order to compute the two velocity increments.

3. The time at the first rocket firing.

4. The lighting conditions at landing.

5. The total fuel consumed in the rocket firings.

6. The distribution of fuel usage between the first and second rocket firings.

7. The degree of preference for one landing site over another.

8. The perigee altitude of the transfer trajectory.

9. The radiation received during the return.

It is evident that the choice of an abort trajectory is affected by many interacting considerations and this decision is then too complex for on-board processing. Consequently, the landing site and its associated abort trajectory must be selected before flight on the basis of a nominal outbound trajectory. Therefore, an extensive preflight analysis will be required.

In the preflight analysis, several points, spaced a few hours apart on the nominal outbound trajectory, are selected and the abort problem is carefully examined for each point. In the preflight study many return trajectories are considered and a "best" abort trajectory from each selected point on the outbound reference trajectory is determined. It should be emphasized that this procedure does not preclude a single-impulse return. If at a particular point the various conditions occur which theoretically permit a single-impulse return to the desired landing site, this trajectory will be used if it is best.

This simplifying concept of aborting from preselected way stations has been used previously by Kelley (ref. 5) and by Callas (ref. 6) to reduce the complexity of the decisions needed in an emergency. However, in this report a way station is defined as a preselected range on the actual outbound trajectory rather than as a particular point on a nominal trajectory.

SIMPLIFYING ASSUMPTIONS

Physical Simplifications

1. Rocket thrust is considered to be an impulse.

Predicting the motion of a space vehicle is very difficult because it is influenced by an asymmetrical earth, the moon, and the sun which are in specific positions at specific times. This task must be eased substantially to enable the astronaut to accomplish it on board. In order to minimize the required computations it will be necessary to use impulsive rocket firings and two-body trajectories. The rocket firings can be assumed to be impulsive because they will occur over very short periods compared to the period of the orbit.

2. The significant gravitational force is from the earth and the gravitational forces of the sun and moon may be neglected.

Of all trajectories leaving the earth, the circumlunar trajectories are, naturally, the most influenced by the third body, so it is sufficient to examine them. For a typical circumlunar trajectory, it is known (ref. 2) that two-body dynamics is not enough to insure a safe entry with a single-impulse return. However, if the two-body correction is made, the errors introduced by the inaccurate guidance can be corrected, at a range of 20,000 km from the earth, with a penalty of less than 100 m/sec. This is true under the reasonable restrictions of an abort range less than 200,000 km and a first velocity increment greater than 1 km/sec. Since the Apollo space vehicle will have an abort capability of more than 3 km/sec (ref. 7), 100 m/sec is not worrisome from a fuel standpoint. Other interplanetary vehicles may not have as large an abort capability but they presumably will not have a trajectory as much affected by the moon. Thus the use of two-body trajectories is satisfactory in either case.

3. The earth is considered to be spherical and homogeneous.

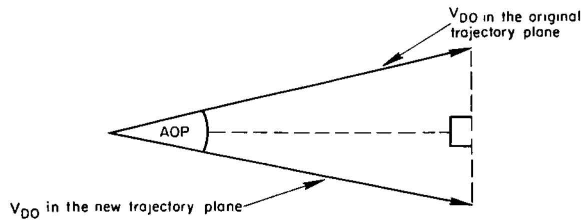
The effects of the oblateness of the earth and its uneven composition are very small if the abort range is greater than 2 earth radii, and these effects are always of less significance than the effects produced by the aiming errors associated with the application of the first velocity increment.

Operational Simplifications

A landing site and the time to land are selected before flight for each way station on the nominal outbound trajectory. In order to determine the required abort maneuver the astronaut must know the estimated position, velocity, and time at the way station. Only a minimal burden is imposed on the normal navigation system if the estimated $\overline{R}$, $\overline{V}$, T of the next one or two way stations is made routinely available to the crew since the aborts will be made at way stations known in advance and since the on-board computer in its routine navigation function is capable of predicting the seven-dimensional state vector $\overline{R}$, $\overline{V}$, T. The routine possession of this information simplifies decision making when an emergency develops.

There are two velocity corrections to be determined but the second correction is used not only for a nominal maneuver but also to correct the errors that build up as a consequence of various earlier errors, including the imperfect achievement of the first rocket firing. Therefore, while the first on-board calculation must produce the three orthogonal components of the velocity to be added in the first velocity increment, it need only determine the magnitude of the second velocity increment. This magnitude is needed so that the total fuel consumption will be known to be within available limits. This report is only concerned with developing the data necessary before the astronaut makes the first correction; therefore, the components of the second velocity correction will not be calculated.

An abort will normally be accomplished with the first impulse applied from a few minutes to an hour after the decision to abort is made and the second impulse applied sufficiently near the earth to assure safe entry. The plane change maneuver can be accomplished with the first rocket firing alone, or the second alone, or partially with each firing. Consider the velocity increment penalty caused by a plane change where V_{DO}, the velocity downrange and perpendicular to the radius vector at the time the plane change is initiated, does not change in magnitude. This situation is illustrated in sketch (a) and it is evident that the velocity increment penalty caused by the plane change is $2V_{DO}[\sin(AOP/2)]$. When the magnitude of the downrange velocity does change, the penalty caused by the plane change may be equal to or less than this.



Sketch (a)

Just before the second impulse is applied near the earth, V_D will ordinarily be substantially greater than the V_D just before the first impulse since in free fall V_D increases monotonically as range decreases. Since the velocity increment penalty is proportional to V_D, it is generally more efficient to accomplish the total plane change with the first rocket firing. This will be the technique used here. This technique, making the total plane change with the first impulse, has the advantage of being simplest analytically also.

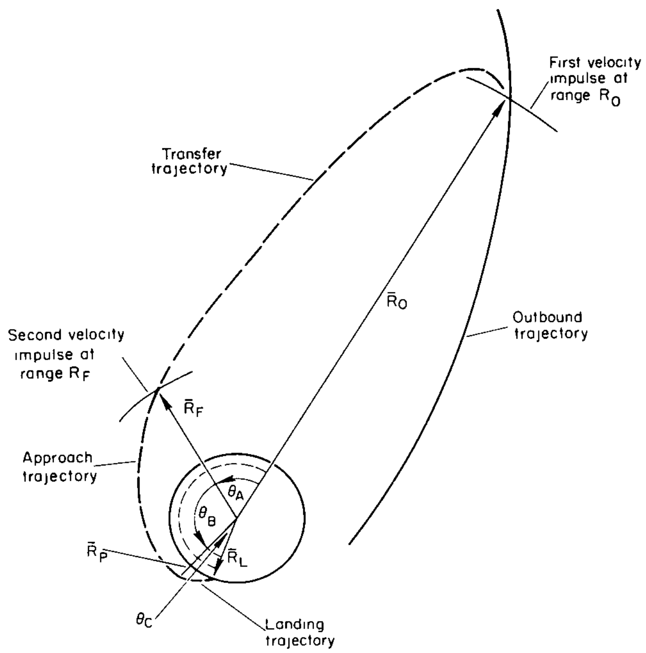


Figure 1.- Geometry for a typical abort.

The astronauts will have available on board a data table of pertinent information about a desirable approach trajectory. A typical abort trajectory as shown in figure 1, may be divided into three parts: the transfer trajectory from the first velocity impulse to the second, the approach trajectory from the second velocity impulse to vacuum perigee, and the landing trajectory from perigee to landing. The plane of these trajectories is defined by the locations of the landing site at the time of landing and the abort point. This latter location is obtained from the on-board computer, as mentioned earlier, but the location and time of landing, X_L, Y_L, Z_L, T_L, should be known beforehand.

Consider now the approach portion of the abort trajectory and a fictitious vehicle moving in the proper plane on an approach trajectory which will land at the desired time and place. The basic problem considered here is to rendezvous with this fictitious vehicle; having done this a successful landing will follow. Pertinent characteristics of the approach trajectory of this vehicle are:

a. The rendezvous radius and time, R_F and T_F.

b. The velocity components of the fictitious vehicle at that radius, V_{RF} and V_{DF}.

c. The perigee of the approach trajectory, R_P.

d. The central angles involved, θ_B and θ_C.

This information about the nominal abort trajectory, that is, the landing site and the characteristics of the approach trajectory, will be used by the astronaut in computing the first velocity correction. This information can be tabulated separately for each way station, R_O. Each table will then contain R_O, X_L, Y_L, Z_L, T_L, R_F, T_F, V_{RF}, V_{DF}, R_P, θ_B, and θ_C.

MATHEMATICS

It now remains to determine what equations are necessary and to write them in that form which is easiest to solve. The conic trajectory simplifications and the plane change simplifications allow a complete description of the transfer trajectory (fig. 1); no description is needed for the approach and landing trajectories since their general characteristics are known. The following five equations describe everything about the transfer trajectory except the time required to traverse it. The time increment equation, for elliptic orbits, will be discussed later.

$$V_{R_{req}} = \frac{\mu(1 - \cos\theta_A)}{h \sin\theta_A} - \frac{h}{R_O \sin\theta_A}\left(\frac{R_O}{R_F} - \cos\theta_A\right) \tag{1}$$

$$h = R_O V_{D_{req}} \tag{2}$$

$$V_{R_{req}}\bar{U}_R + V_{D_{req}}\bar{U}_D = \bar{V}_{T_{req}} \tag{3}$$

$$-(V_{RF} - \Delta V_{R2}) = \frac{\mu(1 - \cos\theta_A)}{h \sin\theta_A} - \frac{h}{R_O \sin\theta_A}\left(1 - \frac{R_O}{R_F}\cos\theta_A\right) \tag{4}$$

where

$$V_{RF} - \Delta V_{R2} = V_{R\beta}$$

$$h = R_F(V_{DF} - \Delta V_{D2}) = R_F V_{D\beta} \tag{5}$$

Equations (1) and (4) are obtained by forming the vector product of equation (12) of reference 8 with the unit vector in the appropriate radial direction. Equations (2) and (5) are statements concerning the constant angular momentum of the transfer trajectory.

These equations are quite simple and some of the terms in equation (1) are used again in equation (4). The four velocities to be determined $V_{R_{req}}$, $\Delta V_{D_{req}}$, ΔV_{R2}, ΔV_{D2} are expressed in equations (1), (2), (4), and (5) in terms of quantities known at the abort point and θ_A and h, the true anomaly increment and the angular momentum.

With θ_A and h known, equations (1) and (2) determine the transfer trajectory. Equation (3) expresses the total velocity required at the abort point in terms of the results of equations (1) and (2) and the unit vectors in the radial and new downrange directions. These unit vectors are:

$$\bar{U}_R = \bar{R}_O/R_O \tag{6}$$

$$\bar{U}_D = \pm[(\bar{R}_O \times \bar{R}_L)/|\bar{R}_O \times \bar{R}_L|] \times \bar{U}_R \tag{7}$$

The components of the first velocity increment to be added are simply obtained by subtracting vectorially the initial velocity at the abort point from the desired velocity at the abort point, $\bar{V}_{T_{req}}$. Equations (4) and (5) are needed to obtain the magnitude of the second velocity increment.

The incremental true anomaly, θ_A, is available from the next equations:

$$\frac{\overline{R_O} \cdot \overline{R_L}}{R_O R_L} = \cos(s) \qquad \text{where} \qquad s < 180^\circ \tag{8}$$

$$ANG = \begin{Bmatrix} s \\ 2\Pi - s \end{Bmatrix} \qquad \text{if} \qquad (\bar{R}_O \times \bar{V}_O) \cdot (\bar{R}_O \times \bar{R}_L) \qquad \text{is} \qquad \begin{Bmatrix} + \\ - \end{Bmatrix} \tag{9}$$

$$\theta_A = ANG - \theta_B - \theta_C \tag{10}$$

The quantity ANG, which is the true anomaly downrange from the abort point to the landing site, is two-valued in equation (9). This equation determines whether ANG is more or less than 180° and the proper values in equation (9) are selected before flight by computation of the vector product along the nominal abort trajectory. This choice is included in the data table since only a most extreme trajectory deviation, such as a gross misfire on injection, could affect this choice. In such an extreme circumstance the crew would probably use a minimum time abort to any safe entry and not attempt to obtain a desirable landing site.

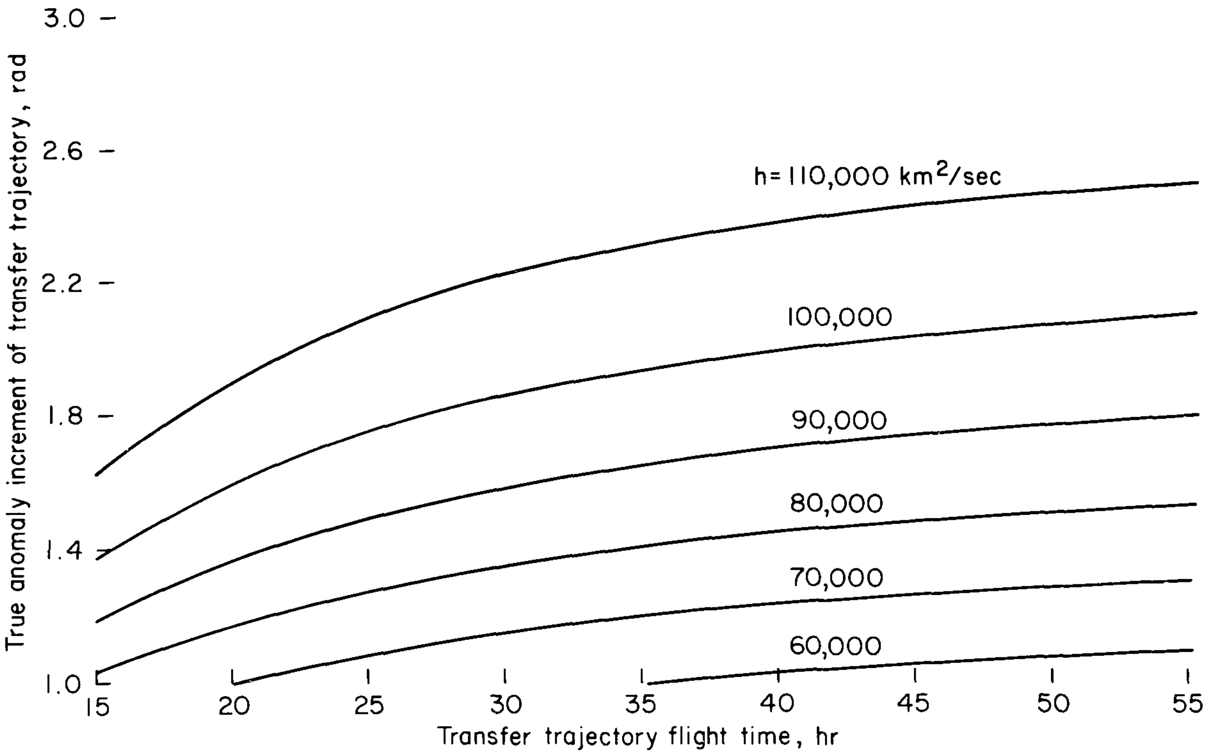
Now it only remains to find the value of angular momentum, h, that correlates with the known time increment which is the difference between the time of rendezvous with the fictitious vehicle and the estimated time of arrival at the way station. The following transcendental equation (ref. 9) gives the transfer time

$$T_A = \sqrt{\frac{a^3}{\mu}} \left[E_\beta - E_{req} - e(\sin E_\beta - \sin E_{req})\right] \tag{11}$$

where E_{req} is the eccentric anomaly immediately after the first velocity increment and E_{β} is the eccentric anomaly just before the second velocity increment. The parameters a, E_{req}, E_{β} are determinable in terms of quantities known at the abort point and θ_A and h. (See appendix A.) Thus given θ_A and h equation (11) will yield the transfer time. This time equation must be solved simultaneously with equations (1) and (2) which determine the velocity increment components $V_{R_{req}}$, $V_{D_{req}}$ and the pertinent equations of appendix A.

The solution of these equations is not straightforward and a numerical solution would require a tedious iterative procedure. The astronaut cannot find the solution to these equations with a practical amount of hand calculation and it would be objectionable to burden the main on-board computer with this task since appreciable additional computer storage and time would be required. This impasse may be avoided by more preflight analysis; these equations are solved for all values of angular momentum, h, and incremental true anomaly, θ_A, which are of interest for a particular way station and a particular terminal range and the solutions are displayed graphically.

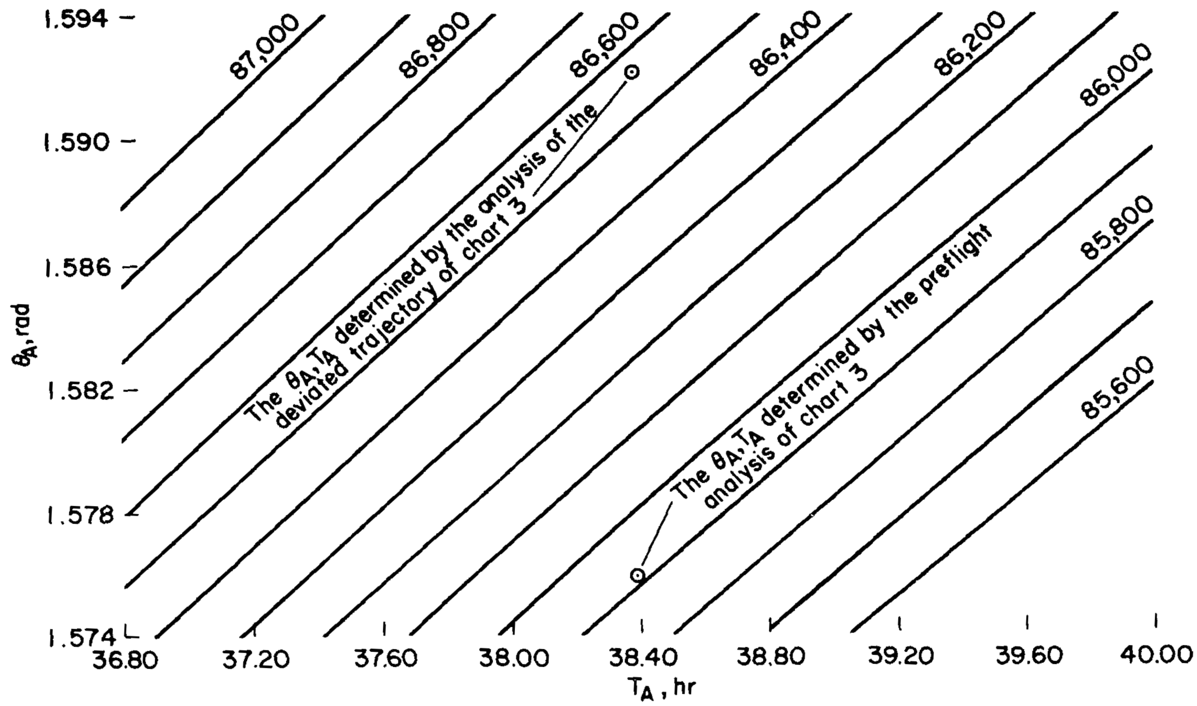
An example of such a graph is shown in figure 2(a) which is an overall view of the relationship between the angular momentum, h, the true anomaly increment, θ_A, and the time increment, T_A. The value of h cannot be



(a) General view.

Figure 2.- Conic data for a range of 150,000 km at abort and a range of 20,000 km at the time of the second velocity correction.

determined with sufficient accuracy from this graph but a section of the figure can be enlarged, as in figure 2(b), so that h can be interpolated to four significant figures.



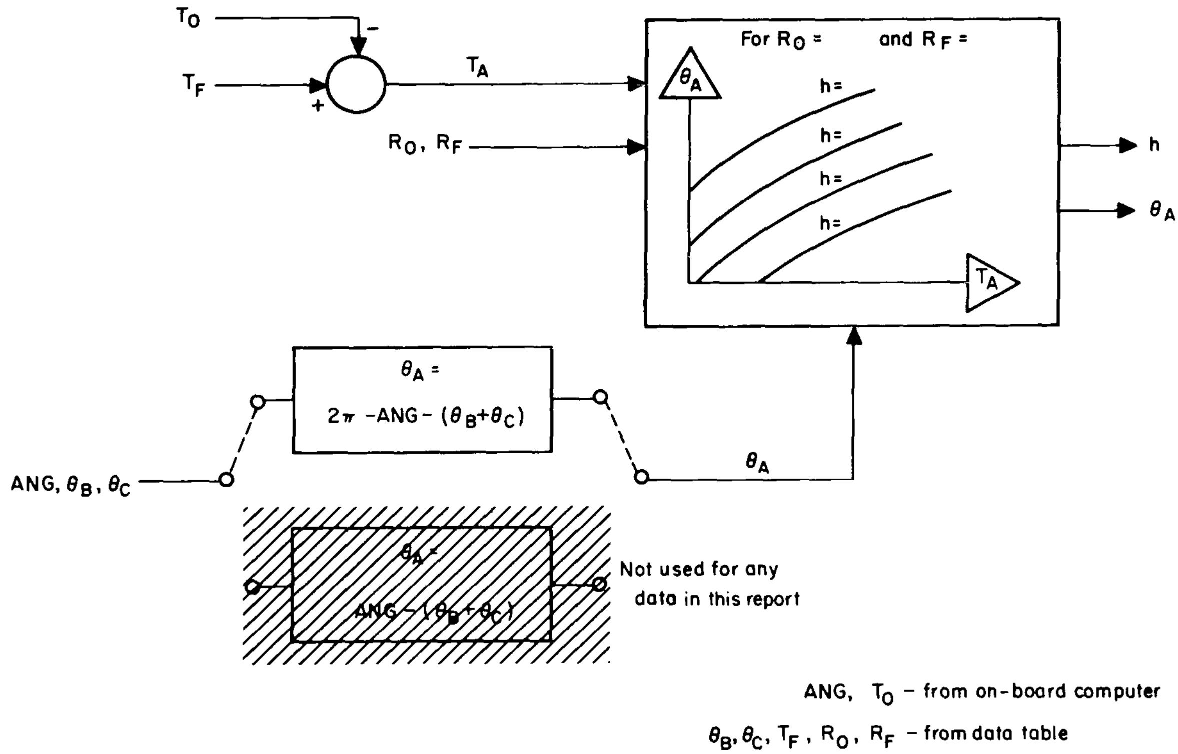
(b) Detailed view.

Figure 2.- Concluded.

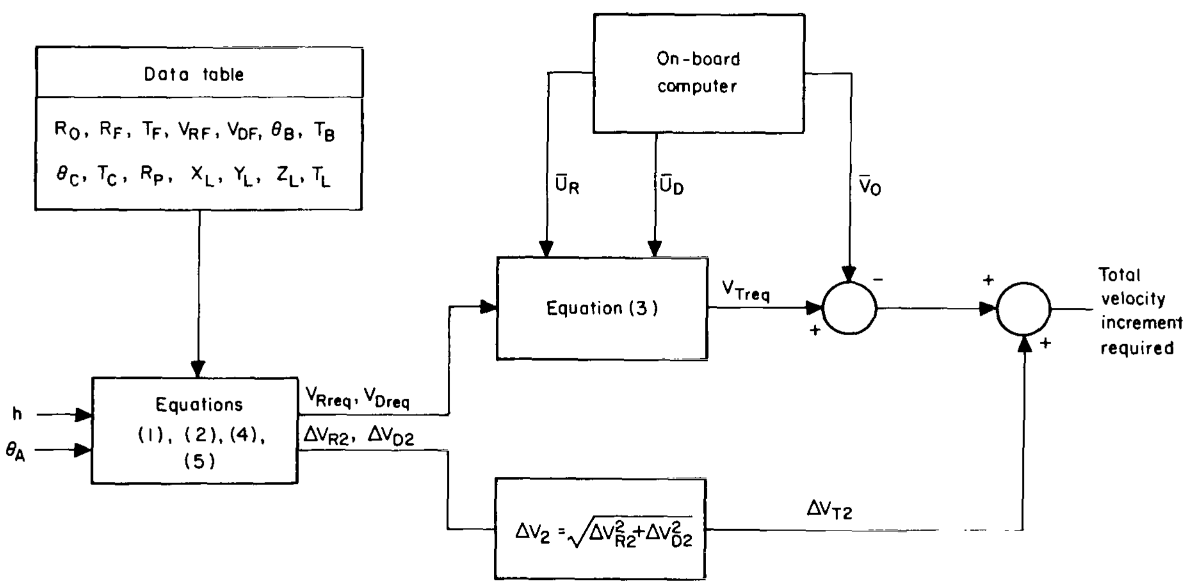
METHOD OF COMPUTATION

With this overview of the relevant equations in mind it is proposed that the on-board computer solve equations (6), (7), and (8) and give the results $\overline{U}_R$, $\overline{U}_D$, and ANG along with the routine projections of $\overline{R}_O$, $\overline{V}_O$, T_O to the way station. This will consume computer time and space but equations (6), (7), and (8) are straightforward so that the additional on-board computer requirement is small. The solution of equation (7) does require that knowledge of the landing site coordinates appropriate to a given way station be available to the on-board computer. The on-board computer subroutine might be manually initiated by the astronaut inserting into the computer the landing site coordinates and the way station range. The quantities $\overline{R}_O$, $\overline{V}_O$, T_O, ANG, $\overline{U}_R$, $\overline{U}_D$ would then be computed and made available to the crew. This information, together with preflight data, is all that is necessary for the two-impulse calculation.

The block diagram presentation of figures 3(a) and 3(b) shows the order in which the computations must proceed. In figure 3(a), the alternate solution for θ_A is shaded since for all the data developed in this study $\theta_A + \theta_B + \theta_C$ was greater than 180°.



(a) First half.



(b) Second half.

Figure 3.- Flow chart for computations.

It should be pointed out that this computational scheme, with or without a reference trajectory, offers a backup mode of operation for calculating mid-course corrections for interplanetary missions when computer failure is more likely because of extended flight time. The graphical solution approximates the numerical integration capability of the main computer for this purpose. If $\overline{R}$, $\overline{V}$, T can be estimated then $\overline{U}_R$, $\overline{U}_D$, etc., can be computed on board with little difficulty. It is, of course, also required that the inertial position coordinates of the target point and a desired time to reach the target point be known.

The on-board numerical procedure can be kept simple conceptually through the use of a computing form which has on it all the numbers pertinent to the solution for the reference abort from the particular way station. Then the number resulting at any stage from the actual on-board computations is entered directly below the corresponding reference trajectory number. This will prevent gross errors since these numbers will normally be similar in sign and magnitude. This is so because ordinarily the actual outbound trajectory is a perturbation of the reference trajectory. The theory, however, is not limited by a small perturbation requirement. A suggested organization of the necessary computations is given in chart 1(a) which shows the form as it appears before flight. The crosshatched areas indicate the places where the data furnished by the on-board computer is entered.

The calculations can be performed in 15 minutes including the sine and cosine lookup and the graphical interpolation. The materials required for this calculation are: a pencil, the reference computation form, sine and cosine tables, the family of graphs for the time constraint, and a small multiplier of at least four digit accuracy. This calculation may be performed after a decision to abort is made or it may be done periodically as a desirable routine navigation procedure just as present day aerial navigators periodically calculate the point of no return in order to expedite decision making in the event of an abort.

Although it is expected that the actual trajectory will be near the reference trajectory this computation will be valid and the space vehicle will land at the desired landing site even if the actual trajectory is very different from that intended. The computation uses Keplerian orbit theory and is independent of the reference trajectory which is used only for determining, in a complex preflight examination, the most desirable landing site for a given abort range.

However, large deviations from the nominal may require additional enlarged sections, similar to those in figure 2(a), to be available. Numerical data on this point are presented later. A further limitation on large trajectory deviations is that the fuel requirements generally increase. When the calculations produce incremental velocity requirements, ΔV_1 and ΔV_2, that are not achievable because of fuel limitations then a return without regard for a landing site may be accomplished. Alternatively, if time is available, another two-impulse calculation may be performed with perhaps a different abort range, a different landing site, or a different landing time.

NUMERICAL RESULTS

In order to demonstrate the flexibility available with two rocket firings, the landing sites chosen for the numerical work were dispersed in latitude including one whose latitude (34° N) is appreciably greater than the inclination of the reference trajectory (28.6°). The latitude of the landing site of an in-plane return cannot exceed the inclination of the actual trajectory plane. Consequently, a return to the 34° latitude landing site will require a plane change if the inclination of the outgoing trajectory plane is less than 34°. The three landing sites are:

Louisiana	94° West,	34° North
Hawaii	158° West,	24° North
Australia	150° East,	15° South

In the future, water landings may not be required and a landing site within the continental United States would reduce the expensive deployment of men and machines currently necessary. The Louisiana site was selected for these reasons. The other two sites were selected because they are secure water areas easily patrolled.

The reference trajectory used in obtaining the data is a typical lunar free return trajectory having a 100 nautical mile perilune and a transit time of 70 hours from injection to perilune. This trajectory, which was also used in obtaining the data of reference 6, is defined further in that report.

Since the objective of this study is only to show the feasibility and the flexibility of on-board manual abort calculations, a detailed optimization of the possible abort trajectories from each of many way stations was not attempted. The last two abort trajectory sections are the same throughout this report. The approach trajectory has an eccentricity of 0.94 with R_F = 20,000 km; V_{RF} = -4.869 km/sec; V_{DF} = +3.526 km/sec; θ_B = 113.6°; T_B = 0.8315 hr. The atmospheric portion of the trajectory, which includes all the landing trajectory and part of the approach trajectory, has an entry range of 2,000 nautical miles from 400,000 ft to landing. Using an approximate linear relation given by Cicolani (ref. 10), between entry time and entry range, the following is readily established:

Range, entry to landing, nautical miles	Time, entry to landing, hr	Time, perigee to landing, hr	θ_C, perigee to landing, radians
2000	0.1910	0.1571	0.3713

This is part of the information available to the crew from preflight analysis.

Data were obtained for only three way stations at distances from the earth of 100, 150, and 200 thousand km. Even without any freedom in selection of the last two trajectory sections, solutions within the capability of the

Apollo vehicle were obtained at each way station for more than one landing site. This is shown in table I which points up the flexibility inherent in a two-impulse return. Successful returns can be made to all three landing sites from all three way stations, but at the 100,000 km station, both the Australia and Louisiana landing sites, require near capacity amounts of fuel for the landing times tabulated. Returns which land 24 hours later require much less fuel.

It might seem at first that the Australian returns would be more efficient if the plane change angle (AOP) were reduced, but this is not true in this particular case. For example, a return to Australia from the 100,000 km way station, with a landing time later than the tabulated case, has the following characteristics:

$$\text{AOP} = 4.4^{\circ} \qquad T_A = 32.66 \text{ hr}$$

$$\Delta V_1 = 1.719 \text{ km/sec} \qquad \Sigma\theta = 304.6^{\circ}$$

$$\Delta V_2 = 4.240 \text{ km/sec}$$

This return trajectory has a smaller plane change angle, but requires considerably more fuel.

No consideration has been given in table I to the lighting conditions at landing since normally one of the dispersed sites available from a given way station will be in daylight. Moreover, a daylight landing may not be required. For Apollo, it is planned that the early trips will be made when the earth side of the moon is in the sun. Then the perigee location for single-impulse midcourse aborts will always be in daylight and, when entry range is limited to 2000 nautical miles, all the attainable landing sites will be in daylight. Under the same entry range restriction two-impulse aborts will not necessarily result in daylight landings since the possible landing areas are greatly increased.

A complete computation for an abort to Hawaii is given in chart 1(b) for the following illustrative situation: the space vehicle is assumed to have arrived at the 200,000 km way station a full hour late but with exactly the $\overline{R}$, $\overline{V}$ of the reference trajectory. This approximates the situation which results from a 1 hour launch delay. It is seen that a satisfactory abort can be accomplished with negligible fuel penalty despite this delay. The nominal velocity increments are $\Delta V_1 = 2.157$ km/sec and $\Delta V_2 = 0.092$ km/sec. With the delay, the velocity increments are $\Delta V_1 = 2.219$ km/sec and $\Delta V_2 = 0.062$ km/sec.

The data pertaining to the reference abort trajectory appear in the top of each compartment in chart 1(b). In the bottom of each compartment are the data from the on-board computation where the quantities estimated by the on-board computer are in crosshatched areas.

The major result of this computation is the velocity vector required at the abort point in order to accomplish the desired landing. It is only necesary to compute the components of this velocity vector to an accuracy of 1 m/sec for the following reasons:

1. The effects of the sun and moon are neglected in these two-body computations.

2. When the rockets are fired the actual velocity components will differ from those intended.

3. The quantities ($\overline{R}$, $\overline{V}$, T) furnished by the main on-board computer are estimates.

4. The computations assume an impulsive velocity increment but the rockets will fire for many seconds.

Approximations 1 and 2 above are both capable of producing errors greater than 1 m/sec and approximations 3 and 4 produce further contributions. Consequently, the number of significant figures carried in the computation should only be enough to obtain accuracies approximating 1 m/sec. In chart 2 the reference abort trajectory, displayed in the top of each compartment, was calculated with at least five significant figures at every step, including the graphical lookup. The abort trajectory, displayed in the bottom of each compartment, was calculated with no more than four significant figures. In this case, the less accurate computation was sufficient to obtain all components of the velocity vector accurate to better than 1 m/sec. A 20-inch slide rule, giving about four significant figures, is probably adequate for the computations but a small mechanical multiplier of five digit accuracy is preferable.

When a spacecraft arrives at a way station, it will have position and velocity deviations from the reference trajectory. The analysis of the velocity increment penalty due to velocity deviations alone is simple since, conceptually, one can fire a rocket whose velocity increment exactly nulls the velocity deviation and thus returns the spacecraft precisely to the reference trajectory. Consequently, this velocity increment penalty can be no more than the nulling velocity increment. However, the magnitude of the nulling velocity increment is small, compared to the more than 1.0 km/sec used in the first velocity impulse of the abort maneuver, since a velocity deviation of 50 m/sec is considered to be a large deviation. Therefore, it is not necessary to present detailed numerical data concerning the velocity increment penalty due to velocity deviations.

However, the position deviation may be substantial although its radial component at the way station is zero, by definition. The computation given in chart 3 compares a reference abort at the 150,000 km way station with an abort from a trajectory which deviated 5,000 km out of the intended orbital plane at the time of arrival at the way station. It is assumed that the spacecraft traveling the deviated trajectory arrived at the way station on the time schedule of the reference trajectory and with the same velocity vector. This comparison demonstrates that this type of error does not necessitate a substantial increase in velocity increment. In this example, figure 2(b) was used to determine the angular momentum demonstrating that the scales chosen for figure 2(b) cover a sufficient range for reasonable errors. The θ_A and T_A of the lower point marked on figure 2(b) were obtained in the preflight calculation of chart 3. The θ_A and T_A of the upper point marked in figure 2(b) were obtained through the calculation related to the deviation trajectory of chart 3.

The family of curves presented in figure 2(b) are very nearly straight lines, suggesting possible future work in determining h from θ_A, T_A. The result would be to replace the graphs of general conic characteristics with a table based on a linear representation of h. That is $h = K_1T_A + K_2$ where K_1, K_2 are tabulated functions of θ_A. Such a table might cover a wide range of deviations simply.

CONCLUSIONS

The purpose of this study has been to increase the astronaut's capability of determining the two-impulse abort maneuver that will return him to a favorable landing site. A computation procedure has been developed which obtains three orthogonal components of the velocity to be added for the first impulsive rocket firing and the magnitude of the second velocity increment. The on-board computations have been organized into a computing form of reasonable simplicity. The three orthogonal components of the first velocity increment can be determined manually to sufficient accuracy in 15 minutes.

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APPENDIX A

PROCEDURE FOR DETERMINING THE TRANSFER TIME

The purpose of this appendix is to show how the time increment required for the transfer trajectory may be determined when the true anomaly increment, θ_A, and the angular momentum, h, are given along with the initial and terminal radii and radial velocities of the transfer trajectory. This time increment is:

$$T_A = \sqrt{\frac{a^3}{\mu}}\,[E_\beta - E_{req} - e(\sin E_\beta - \sin E_{req})] \tag{A1}$$

The following two equations are standard conic relations:

$$e \cos \theta_{req} = \frac{h^2}{\mu R_O} - 1 \tag{A2}$$

$$e \cos(\theta_{req} + \theta_A) = \frac{h^2}{\mu R_F} - 1 \tag{A3}$$

The eccentricity, e, may be found by expanding $\cos(\theta_{req} + \theta_A)$ and using equation (A2) in equation (A3). This yields:

$$e \sin \theta_{req} = \left[\left(\frac{h^2}{\mu R_O} - 1\right)\cos \theta_A - \left(\frac{h^2}{\mu R_F} - 1\right)\right]\frac{1}{\sin \theta_A} \tag{A4}$$

Now if equations (A2) and (A4) are squared and added, the eccentricity is determinable as:

$$e^2 = \left\{\begin{array}{l} [h^2/(\mu R_O) - 1]^2 \\ -2 \cos \theta_A[h^2/(\mu R_O) - 1][h^2/(\mu R_F) - 1] \\ +[h^2/(\mu R_F) - 1]^2 \end{array}\right\}(1/\sin \theta_A)^2 \tag{A5}$$

The radius of perigee, R_P, and the semimajor axis, a, are found from:

$$R_P = \frac{h^2}{\mu(1 + e)}$$

$$a = \frac{R_P}{1 - e}$$

The initial eccentric anomaly of the transfer orbit, E_{req}, is given by:

$$\cos E_{req} = \frac{R_P - R_O(1 - e)}{R_P e}$$

$$E_{req} = \arccos(\cos E_{req}) \qquad \text{for} \qquad 0 < E_{req} < \pi$$

$$E_{req} = \begin{cases} 2\pi - E_{req} & \text{if} \quad V_{R_{req}} < 0 \\ E_{req} & \text{if} \quad V_{R_{req}} > 0 \end{cases}$$

The terminal eccentric anomaly of the transfer orbit, E_β, is given by:

$$\cos E_\beta = \frac{R_P - R_F(1 - e)}{R_P e}$$

$$E_\beta = \arccos(\cos E_\beta) \qquad \text{for} \qquad 0 < E_\beta < \pi$$

$$E_\beta = \begin{cases} 2\pi - E_\beta & \text{if} \quad V_{R_\beta} < 0 \\ E_\beta & \text{if} \quad V_{R_\beta} > 0 \end{cases}$$

In this report V_{R_β} was always less than zero. The time increment, T_A, may now be determined from equation (A1).

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TABLE I.- RESULTS OF ABORT COMPUTATIONS FOR THREE WAY STATIONS AND THREE LANDING SITES WITH A 2000 NAUTICAL MILE ENTRY RANGE

Landing site	Way station at 100,000 km		Way station at 150,000 km		Way station at 200,000 km
Louisiana	AOP = 23.4° ΔV_1 = 2.183 ΔV_2 = 1.044 T_A = 19.84 $\Sigma\theta$ = 218.3°	Time between way stations = 6.4 hours	AOP = 28.3° ΔV_1 = 1.606 ΔV_2 = 1.116 T_A = 38.39 $\Sigma\theta$ = 225.2°	Time between way stations = 7.9 hours	AOP = 30.2° ΔV_1 = 1.382 ΔV_2 = 0.306 T_A = 53.39 $\Sigma\theta$ = 209.4°
Hawaii	AOP = 0.0 ΔV_1 = 0.971 ΔV_2 = 0.472 T_A = 46.81 $\Sigma\theta$ = 204.0°		AOP = 0.0 ΔV_1 = 2.464 ΔV_2 = 0.463 T_A = 40.43 $\Sigma\theta$ = 197.2°		AOP = 0.0 ΔV_1 = 2.157 ΔV_2 = 0.092 T_A = 32.49 $\Sigma\theta$ = 193.9°
Australia	AOP = 74.4° ΔV_1 = 2.005 ΔV_2 = 1.025 T_A = 25.16 $\Sigma\theta$ = 224.3°		AOP = 70.3° ΔV_1 = 1.530 ΔV_2 = 0.918 T_A = 43.21 $\Sigma\theta$ = 223.8°		AOP = 74.2° ΔV_1 = 1.335 ΔV_2 = 0.900 T_A = 59.21 $\Sigma\theta$ = 222.6°

Velocities in km/sec. Time in hours.

Abort from 200,000 km to Hawaii

1	2	3	4	5	6	7	8	9	10	11	12
$\theta_A+\theta_B$	$\theta_A+\theta_B$	θ_B	θ_A	T_A	h	cos θ_A	sin θ_A	$\frac{R_O}{R_F}$	μ/h	1.0 -cos θ_A	
5.9119 -ANG	~~ANG -0.3713~~		1 - 3 or 2 - 3	T_F-T_O	from graph				μ/6	1.0 -7	10 × 11
3.0138		1.9826	1.0312	32.490	69,200	0.5140	0.8578	10.0	5.760	0.4860	2.799
		1.9826						10.0			

13	14	15	16	17	18	19	20	21	22	23	24
R_O/R_F -cos θ_A		$\frac{h}{R_O \sin\theta_A}$	$\frac{\mu(1-\cos)}{h \sin\theta_A}$	$V_{R_{req}}$	$V_{D_{req}}$	1.0 $-R_O$ cos/R_F	V_{RF} +16		ΔV_{R2}	ΔV_{D2}	ΔV_2
9-7		18/8	12/8	16 - 15 ×13	6/R_O	1.0 -9×7		15 × 19	20 - 21	V_{DF} $-h/R_F$	$\sqrt{22^2 + 23^2}$
9.486		0.4034	3.263	-0.5637	0.3460	-4.140	-1.606	-1.670	0.064	0.066	0.092

	$\bar{U}_{RAD}$	$\bar{U}_{DOWN}$	$(V_{R_{req}})\bar{U}_{RAD}+(V_{D_{req}})\bar{U}_{DOWN} = \bar{V}_{T_{req}}$	$\bar{V}_O$	$\bar{V}_{T_{req}}-\bar{V}_O$	$(\bar{V}_T-\bar{V}_O)^2$	ΔV_1^2	ΔV_1	$\bar{R}_L$	$\bar{R}_O$
X_{comp}	-0.1994	0.9422	(0.1124) + (0.3260) = 0.4384	+0.0214	+0.4170	0.1739			-211	-39,887
Y_{comp}	-0.8656	-0.3030	(0.4879) + (-0.1048) = 0.3831	-1.4828	+1.8659	3.4816	4.6511	2.157	5823	-173,130
Z_{comp}	-0.4592	0.1618	(0.2589) + (0.0560) = 0.3149	-0.6829	+0.9978	0.9956			2594	-91,849
									6378	200,000

X_{comp}	.	.	(.) + (.) = .	.	.	.				
Y_{comp}	.	.	(.) + (.) = .	.	.	.	.	.		
Z_{comp}	.	.	(.) + (.) = .	.	.	.				

ANG = 2.8981	T_O = 33.601	T_F = 66.091	T_L = 67.080	T_C = 0.157
ANG = .	T_O = .			

μ = 398,600 km^3/sec^2 $\qquad$ V_{RF} = -4.8694 km/sec $\qquad$ T_B = 0.832 hr

R_F = 20,000 km $\qquad$ V_{DF} = 3.5258 km/sec $\qquad$ ecc = 0.9400

(a) As it appears before flight.

Chart 1.- The computing form used for an abort from 200,000 km to Hawaii.

Abort from 200,000 km to Hawaii

1	2	3	4	5	6	7	8	9	10	11	12
$\theta_A+\theta_B$	$\theta_A+\theta_B$	θ_B	θ_A	T_A	h	cos θ_A	sin θ_A	$\frac{R_0}{R_F}$	μ/h	1.0 -cos θ_A	
5.9119 -ANG	ANG -0.3713		1-3 or 2-3	T_F-T_0	from graph				μ/6	1.0 -7	10 × 11
3.0138		1.9826	1.0312	32.490	69,200	0.5140	0.8578	10.0	5.760	0.4860	2.799
3.0138		1.9826	*1.0312*	*31.490*	*69,820*	*0.5140*	*0.8578*	10.0	*5.709*	*0.4860*	*2.775*

13	14	15	16	17	18	19	20	21	22	23	24
R_0/R_F -cos θ_A		$\frac{h}{R_0 \sin \theta_A}$	$\frac{\mu(1-\cos)}{h \sin \theta_A}$	$V_{R_{req}}$	$V_{D_{req}}$	1.0 $-R_0$ cos$/R_F$	V_{RF} + 16		ΔV_{R2}	ΔV_{D2}	ΔV_2
9 - 7		18 / 8	12 / 8	16 - 15 × 3	6 $/R_0$	1.0 - 9 × 7		15 × 19	20 - 21	V_{DF} $-h/R_F$	$\sqrt{22^2 + 23^2}$
9.486		0.4034	3.263	-0.5637	0.3460	-4.140	-1.606	-1.670	0.064	0.066	0.092
9.486		*0.4070*	*3.235*	*-0.6258*	*0.3491*	*-4.140*	*-1.634*	*-1.685*	*0.051*	*0.035*	*0.062*

	$\bar{U}_{RAD}$	$\bar{U}_{DOWN}$	$(V_{R_{req}})\bar{U}_{RAD}+(V_{D_{req}})\bar{U}_{DOWN} = \bar{V}_{T_{req}}$	$\bar{V}_0$	$\bar{V}_{T_{req}}-\bar{V}_0$	$(\bar{V}_T-\bar{V}_0)^2$	ΔV_1^2	ΔV_1	$\bar{R}_L$	$\bar{R}_0$
X_{comp}	-0.1994	0.9422	(0.1124) + (0.3260) = 0.4384	0.0214	+0.4170	0.1739			-211	-39,887
Y_{comp}	-0.8656	-0.3030	(0.4879) + (-0.1048) = 0.3831	-1.4828	+1.8659	3.4816	4.6511	2.157	5823	-173,130
Z_{comp}	-0.4592	0.1618	(0.2589) + (0.0560) = 0.3149	-0.6829	+0.9978	0.9956			2594	-91,849
									6378	200,000

X_{comp}	*-0.1994*	*0.9422*	*(0.1248) + (0.3289) = 0.4537*	*0.0214*	*+0.4323*	*0.1869*				*-39,887*
Y_{comp}	*-0.8656*	*-0.3030*	*(0.5417) + (-0.1058) = 0.4359*	*-1.4828*	*+1.9187*	*3.6814*	*4.9226*	*2.219*		*-173,130*
Z_{comp}	*-0.4592*	*0.1618*	*(0.2874) + (0.0565) = 0.3439*	*-0.6829*	*+1.0268*	*1.0543*				*-91,849*

ANG = 2.8981	T_0 = 33.601	T_F = 66.091	T_L = 67.080	T_C = 0.157
ANG = 2.8981	*T_0 = 34.601*			

μ = 398,600 km³/sec² $\quad$ V_{RF} = -4.8694 km/sec $\quad$ T_B = 0.832 hr

R_F = 20,000 km $\quad$ V_{DF} = 3.5258 km/sec $\quad$ ecc = 0.9400

(b) The computing form as it appears after on-board entries are made (launch delay example).

Chart 1.- Concluded.

Abort from 200,000 km to Australia

1	2	3	4	5	6	7	8	9	10	11	12
$\theta_A+\theta_B$	$\theta_A+\theta_B$	θ_B	θ_A	T_A	h	cos θ_A	sin θ_A	$\frac{R_O}{R_F}$	μ/h	1.0 −cos θ_A	
5.9119 −ANG	~~ANG −0.3713~~		(1)−(3) or (2)−(3)	T_F-T_O	from graph				μ/(6)	1.0 −(7)	(10) × (11)
3.5139		1.9826	1.5313	59.209	84290	0.03949	0.99922	10.0	4.7289	0.96051	4.5422
3.514		1.983	*1.531*	*59.21*	*84280*	*0.0398*	*0.9992*	10.0	*4.729*	*0.9602*	*4.541*

13	14	15	16	17	18	19	20	21	22	23	24
R_O/R_F −cos θ_A		$\frac{h}{R_O \sin\theta_A}$	$\frac{\mu(1-\cos)}{h \sin\theta_A}$	$V_{R_{req}}$	$V_{D_{req}}$	1.0 $-R_O \cos/R_F$	V_{RF} + (16)		ΔV_{R2}	ΔV_{D2}	ΔV_2
(9)−(7)		(18)/(8)	(12)/(8)	(16)−(15) ×(13)	(6)/R_O	1.0 −(9)×(7)		(15)×(19)	(20)×(21)	V_{DF} $-h/R_F$	$\sqrt{(22)^2+(23)^2}$
9.9605		0.42178	4.5457	0.34456	0.42145	0.60510	−0.32370	0.25522	−0.57892	−0.68870	0.900
9.960		*0.4217*	*4.545*	*0.3449*	*0.4214*	*0.6020*	*−0.3240*	*0.2539*	*−0.5779*	*−0.6880*	*0.898*

	$\bar{U}_{RAD}$	$\bar{U}_{DOWN}$	$(V_{R_{req}})\bar{U}_{RAD}+(V_{D_{req}})\bar{U}_{DOWN} = \bar{V}_{T_{req}}$	$\bar{V}_O$	$\bar{V}_{T_{req}}-\bar{V}_O$	$(\bar{V}_T-\bar{V}_O)^2$	ΔV_1^2	ΔV_1	$\bar{R}_L$	$\bar{R}_O$
X_{comp}	−0.19944	0.01685	(−0.06872) + (0.00710) = −0.0616	+0.0214	−0.0830	0.007			864	−39,887
Y_{comp}	−0.86563	−0.47162	(−0.29826) + (−0.19876) = −0.4970	−1.4828	+0.9858	0.972	1.782	1.335	6100	−173,130
Z_{comp}	−0.45924	0.88164	(−0.15824) + (0.37157) = 0.2133	−0.6829	+0.8962	0.803			−1651	−91,849
									6378	200,000
X_{comp}	*−0.1994*	*0.0168*	*(−0.0688) + (0.0071) = −0.0617*	*+0.0214*	*−0.0831*	*0.007*				*−39,890*
Y_{comp}	*−0.8656*	*−0.4716*	*(−0.2985) + (−0.1987) = −0.4972*	*−1.4828*	*+0.9856*	*0.971*	*1.781*	*1.335*		*−173,100*
Z_{comp}	*−0.4592*	*0.8816*	*(−0.1584) + (0.3715) = 0.2131*	*−0.6829*	*+0.8960*	*0.803*				*−91,850*

ANG = 2.3980	T_O = 33.601	T_F = 92.810	T_L = 93.799	T_C = 0.157
ANG = 2.398	*T_O = 33.60*			

μ = 398,600 km^3/sec^2 V_{RF} = −4.8694 km/sec T_B = 0.832 hr

R_F = 20,000 km V_{DF} = 3.5258 km/sec ecc = 0.9400

The first calculation used at least five significant figures at every step.
The second calculation used no more than four significant figures.

Chart 2.- Illustrating the effects of computational precision.

Abort from 150,000 km to Louisiana

1	2	3	4	5	6	7	8	9	10	11	12
$\theta_A + \theta_B$	$\theta_A + \theta_B$	θ_B	θ_A	T_A	h	cos θ_A	sin θ_A	$\frac{R_O}{R_F}$	μ/h	1.0 -cos θ_A	
5.9119 -ANG	ANG -0.3713		(1)-(3) or (2)-(3)	$T_F - T_O$	from graph				μ/(6)	1.0 -(7)	(10) × (11)
3.5585		1.9826	1.5759	38.388	85,920	-0.00510	0.99999	7.50	4.6392	1.0051	4.6629
3.5747		1.9826	*1.5921*	*38.388*	*86,570*	*-0.02131*	*0.99977*	7.50	*4.6044*	*1.0213*	*4.7025*

13	14	15	16	17	18	19	20	21	22	23	24
R_O/R_F -cos θ_A		$\frac{h}{R_O \sin \theta_A}$	$\frac{\mu(1-\cos)}{h \sin \theta_A}$	$V_{R_{req}}$	$V_{D_{req}}$	1.0 $-R_O$ cos/R_F	V_{RF} + (16)		ΔV_{R2}	ΔV_{D2}	ΔV_2
(9)-(7)		(18)/(8)	(12)/(8)	(16) - (15) × (13)	(6)/R_O	1.0 -(9)×(7)		(15) × (19)	(20) - (21)	V_{DF} $-h/R_F$	$\sqrt{(22)^2 + (23)^2}$
7.5051		0.57281	4.6629	0.36390	0.57280	1.0390	-0.2093	0.5956	-0.8049	-0.7732	1.116
7.5213		*0.57726*	*4.7036*	*0.36185*	*0.57713*	*1.1598*	*-0.1658*	*0.6695*	*-0.8353*	*-0.8027*	*1.158*

	$\bar{U}_{RAD}$	$\bar{U}_{DOWN}$	$\left(V_{R_{req}}\right)\bar{U}_{RAD} + \left(V_{D_{req}}\right)\bar{U}_{DOWN} = \bar{V}_{T_{req}}$	$\bar{V}_O$	$\bar{V}_{T_{req}} - \bar{V}_O$	$\left(\bar{V}_T - \bar{V}_O\right)^2$	ΔV_1^2	ΔV_1	$\bar{R}_L$	$\bar{R}_O$
X_{comp}	-0.2629	0.9393	(-0.0957) + (0.5380) = 0.4423	-0.0655	+0.5078	0.258			-3066	-39,430
Y_{comp}	-0.8441	-0.1132	(-0.3072) + (-0.0648) = -0.3720	-1.8042	+1.4322	2.051	2.561	1.600	4308	-126,610
Z_{comp}	-0.4674	-0.3239	(-0.1701) + (-0.1855) = -0.3556	-0.8571	+0.5015	0.252			3567	-70,110
									6378	150,000

X_{comp}	*-0.2711*	*0.9282*	*(-0.0981) + (0.5357) = 0.4376*	*-0.0655*	*+0.5031*	*0.253*				*-40,662*
Y_{comp}	*-0.8572*	*-0.1123*	*(-0.3102) + (-0.0648) = -0.3750*	*-1.8042*	*+1.4292*	*2.043*	*2.540*	*1.594*		*-128,578*
Z_{comp}	*-0.4379*	*-0.3547*	*(-0.1585) + (-0.2047) = -0.3632*	*-0.8571*	*+0.4939*	*0.244*				*-65,685*

ANG = 2.3534	T_O = 25.668	T_F = 64.056	T_L = 65.045	T_C = 0.157
ANG = 2.3372	*T_O = 25.668*			

$\mu = 398{,}600$ km^3/sec^2 $\quad V_{RF} = -4.8694$ km/sec $\quad T_B = 0.832$ hr

$R_F = 20{,}000$ km $\quad V_{DF} = 3.5258$ km/sec $\quad$ ecc = 0.9400

Chart 3.- Illustrating the effect of a 5,000 km out-of-plane error at the 150,000 km way station.

NASA-Langley, 1968 — 21 A-2694

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